DBX-003-1162002 Seat No. ________

M. Sc. (Sem. II) Examination

July - 2022

Mathematics : CMT-2002

(Complex Analysis)

Faculty Code : 003
Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Each questions carries equal marks.
(3) Figure on the right indicate allotted marks.

1 Answer any **Seven** short questions : **7×2=14**

(1) State : Necessary and sufficient condition for an isolated singularity to be removable singularity.

(2) Let $a, b \in \mathbb{C}$ be fixed, $\gamma : [0,1] \to \mathbb{C}$ be defined as $\gamma(t) = (1-t)a + bt, \forall t \in [0,1], m \geq 0$ be an integer and $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = z^m, \forall z \in \mathbb{C}$, then find $\int_\gamma f$.

(3) Define : (i) Pole (ii) Essential singularity.

(4) Define : Diameter of a set in metric space.

(5) Find $(z, \infty\, 0, 1)$

(6) Define : Smooth and Piecewise smooth path.

(7) Define with example : Analytic function.

(8) State : Leibnitz's rule.

(9) If $(z) = \dfrac{az+b}{cz+d}$, then find $T^{-1}(z)$.

(10) Define : Rectifiable path and length of rectifiable path.

2 Attempt any two : **2×7=14**

(a) State and prove : Taylor's theorem.

(b) State and prove : Fundamental theorem of calculus of line integral.

(c) State without proof : Cauchy's theorem for an open disc and find $\int_{\sigma} \frac{dz}{z^2 - 1}$; where $\sigma(t) = 1 + e^{it}, \forall t \in [0, 2\pi]$.

3 Attempt following both (a) and (b) : **2×7=14**

(a) Find the bilinear transformations taking

(i) $i \to 1, 0 \to \infty, -1 \to 0$

(ii) $1 \to i, 0 \to -i, -1 \to 0$

(b) Find the following :

(i) $\int_{\alpha} \frac{1}{z} dz$; where $\alpha(t) = e^{\text{int}}$ and for all $t \in [0, 2\pi]$.

(ii) $\int_{\gamma} z^n \, dz, \forall n \in \mathbb{Z},$ where $\gamma(t) = e^{it}, \forall t \in [0, 2\pi]$

OR

3 Attempt following : **1×14=14**

(a) State and prove : Necessary and sufficient condition for four distinct points in $\mathbb{C}_{\infty}$ to be on a circle in $\mathbb{C}_{\infty}$

4 Attempt any two **2×7=14**

(a) Let $\gamma : [a,b] \to \mathbb{C}$ be a function of bounded variation, $f : [a,b] \to \mathbb{C}$ be a continuous function and $\{a = t_0 < t_1 < \ldots < t_n = b\}$ be the partition of [a, b]. Then prove that $\int_a^b f \, d\gamma = \sum_{k=1}^{n} \int_{t_{k-1}}^{t_k} f \, d\gamma$

(b) Give an example of the complex function has no primitive. Justify.

(c) Find Laurent's series expansion in the powers of z for $f(z) = \frac{z+2}{z^2 - 2z - 3} in$

(i) $|z| < 1$;

(ii) $1 < |z| < 3$;

(iii) $|z| > 3$.

5 Attempt any two : **7×2=14**

(a) Prove that : Every bilinear transformation can be written as composition of translation, dilation and inversion.

(b) State and prove : Morera's theorem.

(c) It $\gamma : [a,b] \to \mathbb{C}$ is a rectifiable path and $f : \{\gamma\} \to \mathbb{C}$ is continuous then prove that

$$\left|\int_\gamma f\right| < \int_\gamma |f| \, |dz| \le V(\gamma). \sup_{z \in \{\gamma\}} [f(z)].$$

(d) State and prove : Rouche's theorem.